What is the polarization of a wave propagating in the r-direction if its electric field vector at any fixed point in space is given by **(a)** $\mathbf{E}_a = (\mathbf{a}_\theta + \mathbf{a}_\phi j)$ and **(b)** $\mathbf{E}_b = (\mathbf{a}_\theta j + \mathbf{a}_\phi)$? a) LHCP b)RHCP

What is the polarization of a wave radiated along the y-axis, by a z-directed Hertzian dipole? Show that it can be decomposed into left and right circularly polarized waves.

Solution: In the far-field region of a z-directed Hertzian dipole the electri field intensity along the y-axis is given by Eqn (2.8) with $\theta = 90°$

$$\mathbf{E} = \mathbf{a}_\theta j E_0$$

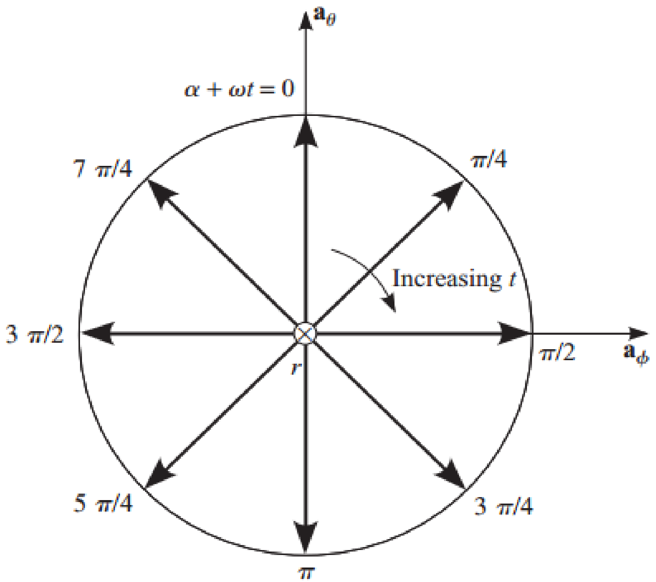
where $E_0 = j\eta k I_0 dl e^{-jkr}/(4\pi r)$. Since it has only a θ-component, it repre sents a linearly polarized wave. Expressing $\mathbf{a}_\theta$ as

$$\mathbf{a}_\theta = \frac{1}{2}[(\mathbf{a}_\theta + \mathbf{a}_\phi j) + (\mathbf{a}_\theta - \mathbf{a}_\phi j)]$$

we can write the electric field intensity as

$$\mathbf{E} = E_0 \frac{1}{2}[(\mathbf{a}_\theta + \mathbf{a}_\phi j) + (\mathbf{a}_\theta - \mathbf{a}_\phi j)]$$

The first term on the right hand side represents a LCP wave and the second term represents a RCP wave.



Electric field vector of a right circularly polarized antenna

Geometrical Arrangement of Small Dipole & Current Distribution

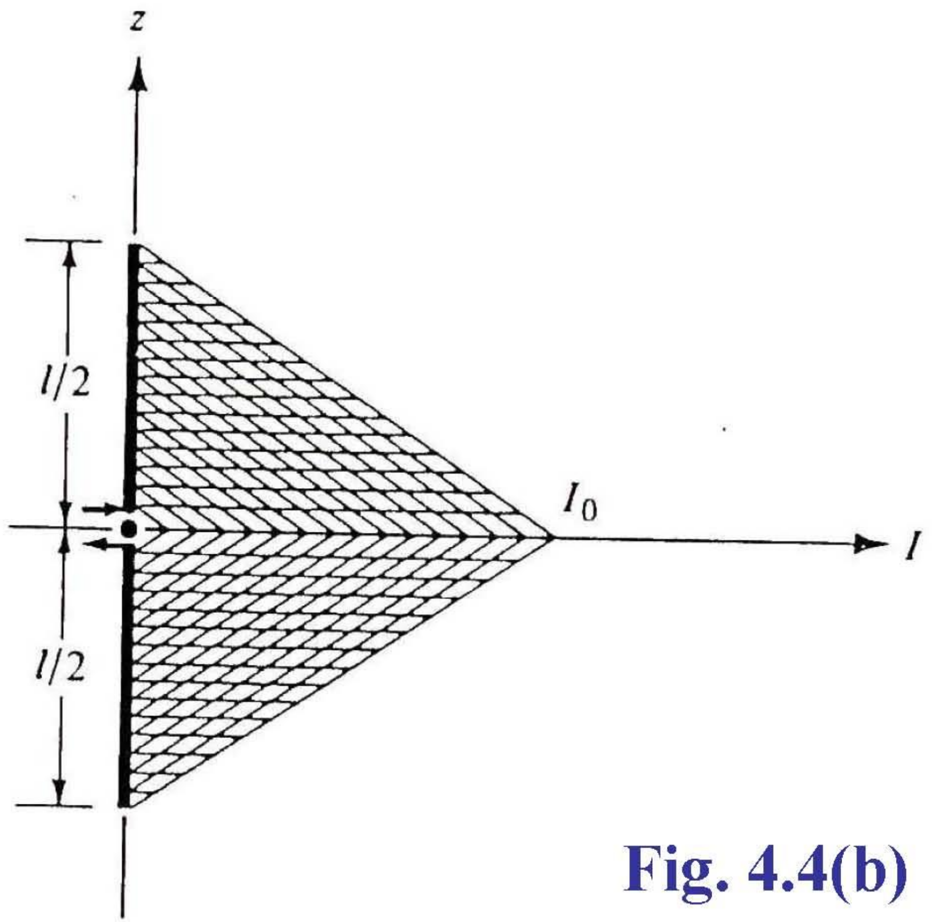


Fig. 4.4(b)

Current distribution of short dipole

$$\underline{I}_e(x', y', z') \simeq \underbrace{\begin{cases} \hat{a}_z I_o\left(1-\frac{2}{l}z'\right) & 0 \le z' \le l/2 \\ \\ \hat{a}_z I_o\left(1+\frac{2}{l}z'\right) & -l/2 \le z' \le 0 \end{cases}}_{\lambda/50 \le l \le \lambda/10} \qquad (4\text{-}33)$$

Vector Potential

$$\underline{A} = \frac{\mu}{4\pi}\int_{-\ell/2}^{\ell/2} \underline{I}_e(z')\frac{e^{-jkR}}{R}d\ell'$$

Approximation: $R \approx r$

$$\underline{A}(x,y,z) = \frac{\mu}{4\pi}\left[\hat{a}_z \int_{-l/2}^{0} I_o\left(1+\frac{2}{l}z'\right)\frac{e^{-jkR}}{R}dz' + \hat{a}_z \int_{0}^{+l/2} I_o\left(1-\frac{2}{l}z'\right)\frac{e^{-jkR}}{R}dz'\right] \quad (4\text{-}34)$$

$$\underline{A}(x,y,z) \simeq \hat{a}_z \frac{1}{2}\left[\frac{l\mu I_o e^{-jkr}}{4\pi r}\right] = \hat{a}_z \frac{1}{2}\left[A_z'\right] \quad (4\text{-}35)$$

$$A_z'(uniform\ current) = \frac{\mu I_o l e^{-jkr}}{4\pi r} = 2A_z(triangular\ current)$$

$$\Rightarrow A_\theta = -A_z \sin\theta = -\frac{\mu I_o \ell}{4\pi r}\sin\theta\ e^{-jkr}$$

$$\Downarrow$$

$$E_\theta \cong -j\omega A_\theta \qquad H_\phi \cong -j\frac{\omega}{\eta}A_\theta = +\frac{E_\theta}{\eta}$$

Far-Field ($kr >> 1$)

$$E_\theta = \frac{1}{2} E_\theta^{'} \simeq j\eta \frac{kI_o l e^{-jkr}}{8\pi r} \sin\theta \qquad (4\text{-}36a)$$

$$H_\phi = \frac{1}{2} H_\phi^{'} \simeq j \frac{kI_o l e^{-jkr}}{8\pi r} \sin\theta \qquad (4\text{-}36c)$$

$$\underbrace{E_r \simeq E_\phi = H_r = H_\theta = 0}_{} \qquad (4\text{-}36b)$$

$$\underline{W}_{rad} = \frac{1}{4} \underline{W}_{rad}^{'}, \quad P_{rad} = \frac{1}{4} P_{rad}^{'}, \quad U_{rad} = \frac{1}{4} U_{rad}^{'}$$

Far-Field ($kr >> 1$)

$$P_{rad} = \frac{1}{2}|I_o|^2 R_r$$

$$R_r = \frac{2P_{rad}}{|I_o|^2} = \frac{2\left(\frac{1}{4}P'_{rad}\right)}{|I_o|^2} = \frac{1}{4}\left(\frac{2P'_{rad}}{|I_o|^2}\right)$$

$$\boxed{R_r = \frac{1}{4}R'_r = 20\pi^2\left(\frac{l}{\lambda}\right)^2} \qquad (4\text{-}37)$$

$$D_o = \frac{4\pi U_{\max}}{P_{rad}} = \frac{4\pi\left(\frac{1}{4}U'_{\max}\right)}{\frac{1}{4}P'_{rad}} = \frac{4\pi U'_{\max}}{P'_{rad}}$$

$$D_o = \frac{4\pi U'_{\max}}{P'_{rad}} = D'_o = \frac{3}{2}$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_o = \frac{\lambda^2}{4\pi} D'_o = \frac{3\lambda^2}{8\pi} = A'_{em}$$

Finite Dipole

Ideal Sinusoidal Current Distribution

$$\underline{I}_e = \begin{cases} \hat{a}_z I_0 \sin\left[k\left(\dfrac{l}{2} - z'\right)\right] & 0 \le z' \le l/2 \\ \hat{a}_z I_0 \sin\left[k\left(\dfrac{l}{2} + z'\right)\right] & -l/2 \le z' \le 0 \end{cases}$$

(4-56)

Current Distributions Along the Length of a Linear Wire Antenna

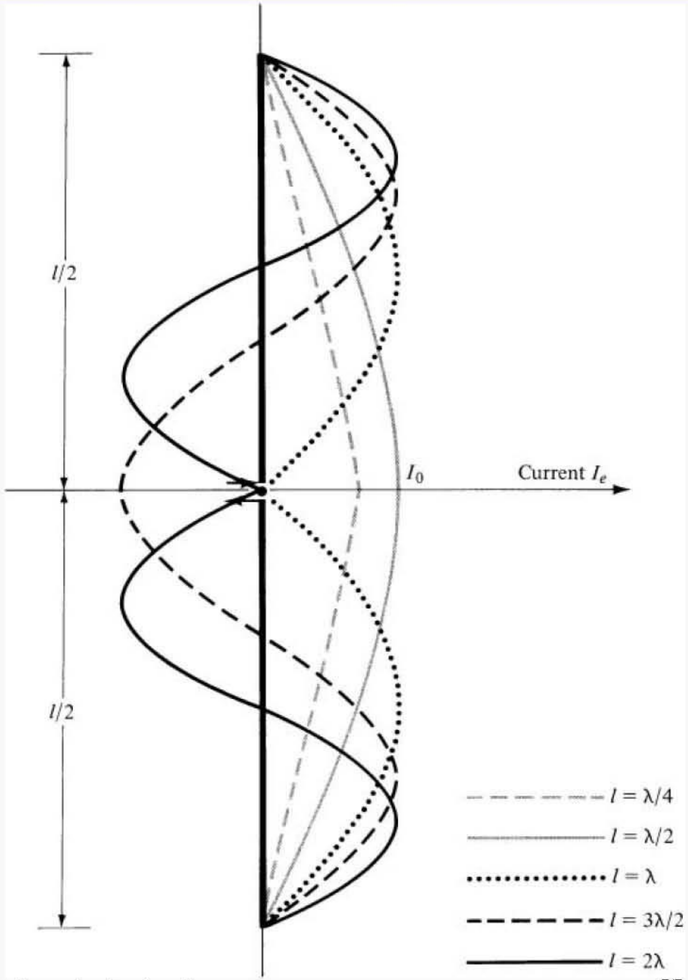


Fig. 4.8

$$E_\theta = j\eta \frac{I_o e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{k\ell}{2}\cos\theta\right)-\cos\left(\frac{k\ell}{2}\right)}{\sin\theta}\right] \quad (4\text{-}62a)$$

$$E_\theta \cong C\left[\underbrace{\frac{\cos\left(\frac{k\ell}{2}\cos\theta\right)-\cos\left(\frac{k\ell}{2}\right)}{\sin\theta}}_{\text{Field Pattern}}\right]$$

$$H_\phi \cong \frac{E_\theta}{\eta}, \qquad C = j\eta \frac{I_o e^{-jkr}}{2\pi r}$$

$$\underline{W}_{av} = \frac{1}{2}\mathrm{Re}\left[\underline{E}\times\underline{H}^*\right] = \frac{1}{2}\mathrm{Re}\left[\hat{a}_\theta E_\theta \times \hat{a}_\phi H_\phi^*\right]$$

$$= \hat{a}_r \frac{1}{2}\mathrm{Re}\left(E_\theta H_\phi^*\right) = \hat{a}_r \frac{1}{2}\mathrm{Re}\left(E_\theta \frac{E_\theta^*}{\eta}\right)$$

$$= \hat{a}_r \frac{1}{2\eta}\mathrm{Re}\left(E_\theta E_\theta^*\right) = \hat{a}_r \frac{1}{2\eta}\mathrm{Re}\left(\left|E_\theta\right|^2\right)$$

$$\underline{W}_{av} = \hat{a}_r \eta \frac{\left|I_0\right|^2}{8\pi^2 r^2}\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]^2 \quad (4\text{-}63)$$

$$W_{av} = W_{rad} = \eta \frac{|I_0|^2}{8\pi^2 r^2} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]^2 \quad \text{(4-63)}$$

$$U_{rad} = r^2 W_{av} = \underbrace{\eta \frac{|I_0|^2}{8\pi^2}}_{B_0} \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]^2 \quad \text{(4-64)}$$

$$U_{rad} = B_0 \underbrace{\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta} \right]^2}_{\text{Power Pattern}}$$

$$R_r = \frac{2P_{rad}}{|I_o|^2}$$

$$R_r = \frac{\eta}{2\pi}\left\{ \begin{array}{l} C + \ln(k\ell) - C_i(k\ell) + \frac{1}{2}\sin(k\ell)\left[S_i(2k\ell) - 2S_i(k\ell)\right] \\ + \frac{1}{2}\cos(k\ell)\left[C + \ln\left(\frac{k\ell}{2}\right) + C_i(2k\ell) - 2C_i(k\ell)\right] \end{array} \right\}$$

(4-70)

$$X_m = \frac{\eta}{4\pi}\left\{ \begin{array}{l} 2S_i(k\ell) + \cos(k\ell)\left[2S_i(2k\ell) - S_i(2k\ell)\right] \\ -\sin(k\ell)\left[2C_i(k\ell) - C_i(2k\ell) - C_i\left(\frac{2ka^2}{\ell}\right)\right] \end{array} \right\}$$

(4-70a)

Directivity

$$D_0 = \frac{4\pi\ U_{\max}}{P_{rad}} = \frac{4\pi\ F(\theta,\phi)\big|_{\max}}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin\theta d\theta d\phi} \quad (4\text{-}71)$$

$$U\Big|_{dipole} = \underbrace{\eta\ \frac{|I_0|^2}{8\pi^2}}_{B_0}\underbrace{\left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right)-\cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]^2}_{F(\theta,\phi)} \quad (4\text{-}72)$$

$$D_o = \frac{2F(\theta)|_{\max}}{Q} \tag{4-75}$$

$$Q = \left\{ \begin{array}{l} C + \ln(k\ell) - C_i(k\ell) + \frac{1}{2}\sin(k\ell)\left[S_i(2k\ell) - 2S_i(k\ell)\right] \\ + \frac{1}{2}\cos(k\ell)\left[C + \ln\left(\frac{k\ell}{2}\right) + C_i(2k\ell) - 2C_i(k\ell)\right] \end{array} \right\} \tag{4-75a}$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_o \tag{4-76}$$

Ideal Sinusoidal Current Distribution

$$\underline{I}_e = \begin{cases} \hat{a}_z I_0 \sin\left[k\left(\dfrac{l}{2} - z'\right)\right] & 0 \le z' \le l/2 \\ \hat{a}_z I_0 \sin\left[k\left(\dfrac{l}{2} + z'\right)\right] & -l/2 \le z' \le 0 \end{cases}$$

(4-56)

Current Distribution of a Linear Wire Antenna

When Current Maximum

Does not Occur at the Input Terminals

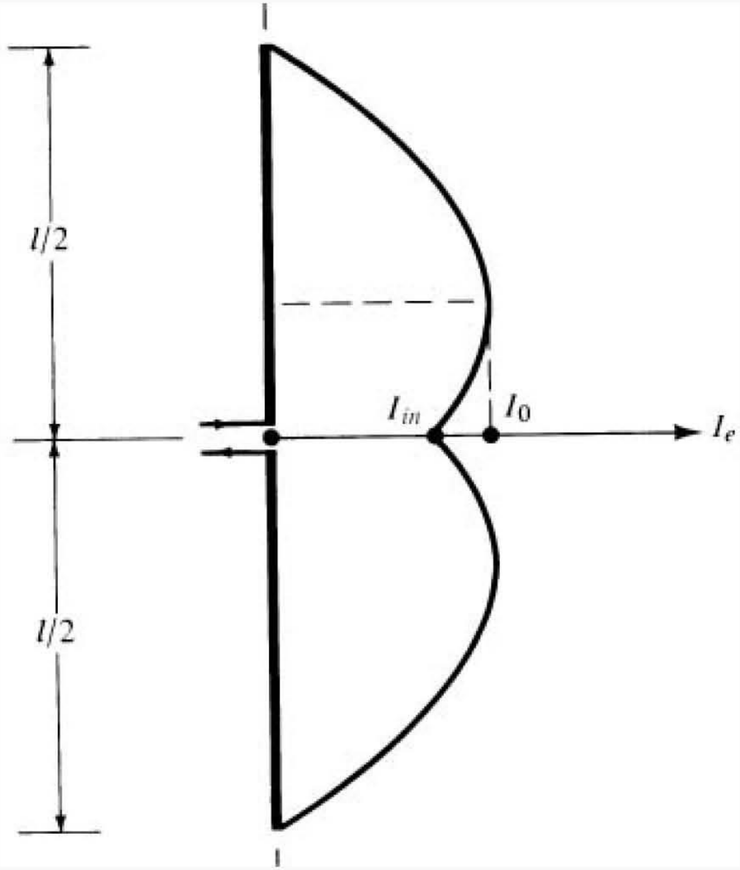


Fig. 4.10

$$P_{rad} = \frac{1}{2}|I_o|^2 R_r = \frac{1}{2}|I_{in}|^2 R_{in} \qquad \text{(4-77)}$$

$$R_{in} = R_r \frac{|I_o|^2}{|I_{in}|^2} = R_r \left|\frac{I_o}{I_{in}}\right|^2 \qquad \text{(4-77a)}$$

$$\underline{I}_e\Big|_{z'=0} = I_o \sin\left[k\left(\frac{\ell}{2} \pm z'\right)\right]\Big|_{z'=0} = I_{in}$$

$$I_{in} = I_o \sin\left(\frac{k\ell}{2}\right) \qquad \text{(4-78)}$$

$$R_{in} = R_r \left| \frac{I_o}{I_{in}} \right|^2 = R_r \left| \frac{I_o}{I_o \sin\left(\frac{k\ell}{2}\right)} \right|^2$$

$$\boxed{R_{in} = \frac{R_r}{\sin^2\left(\frac{k\ell}{2}\right)}} \qquad (4\text{-}79)$$

$$1.\ \ell = \lambda/2: \quad \sin^2\left(\frac{k\ell}{2}\right)\Bigg|_{\ell=\lambda/2} = \sin^2\left(\frac{2\pi}{2\lambda}\frac{\lambda}{2}\right) = \sin^2\left(\frac{\pi}{2}\right) = 1$$

$$R_{in} = R_r$$

$$2.\ \ell = \lambda: \quad \sin^2\left(\frac{k\ell}{2}\right)\Bigg|_{\ell=\lambda} = \sin^2(\pi) = 0$$

$$R_{in} = R_r/0 = \infty$$

Directivity and Radiation/Input Resistance

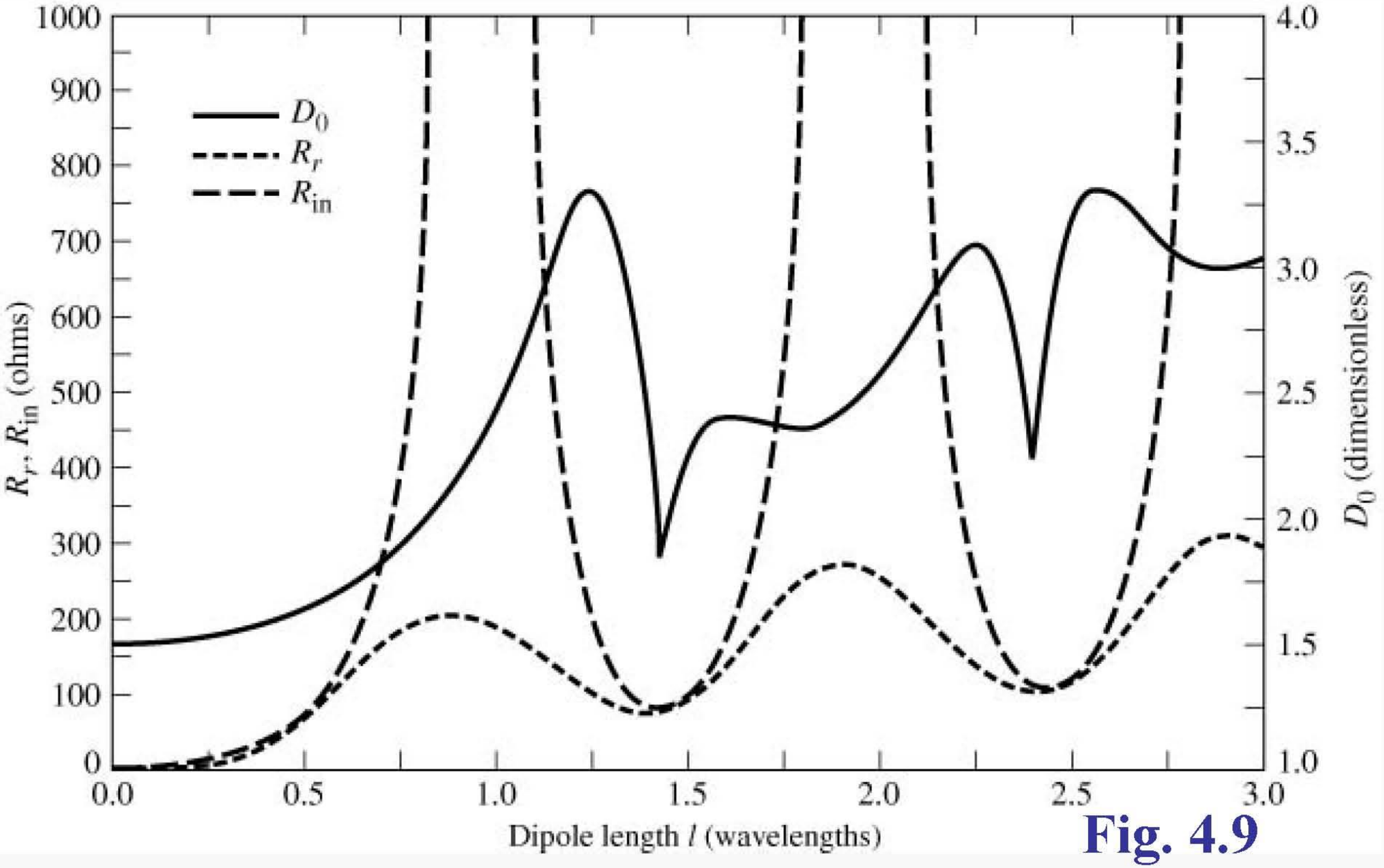


Fig. 4.9

Half-Wavelength Dipole

$(l=\lambda/2)$

Half-Wavelength Dipole ($l = \lambda/2$)

$$E_\theta \simeq j\eta \frac{I_0 e^{-jkr}}{2\pi r}\left[\underbrace{\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}}_{Field\ Pattern}\right] \quad (4\text{-}84)$$

$$H_\phi \simeq j \frac{I_0 e^{-jkr}}{2\pi r}\left[\underbrace{\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}}_{Field\ Pattern}\right] = \frac{E_\theta}{\eta} \quad (4\text{-}85)$$

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = 4\pi \frac{U|_{\theta=\pi/2}}{P_{rad}} = \frac{4}{C_{in}(2\pi)} = \frac{4}{2.435} = 1.643$$

$$C_{in}(2\pi) = 0.577 + \ln(2\pi) - C_i(2\pi)$$
$$= 0.577 + 1.838 - (-0.02) \simeq 2.435 \quad (4\text{-}90)$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 = \frac{\lambda^2}{4\pi}(1.643) = 0.13\lambda^2 \quad (4\text{-}92)$$

$$R_r = \frac{2P_{rad}}{|I_0|^2} \frac{\eta}{4\pi} C_{in}(2\pi) \simeq \frac{120\pi}{4\pi}(2.435) = 73.05 \quad (4\text{-}93)$$

$$X_{in} = X_m = -120 \frac{[\ln(l/2a) - 1]}{\tan(kl/2)}$$

$$a = 10^{-5}\lambda$$

Input Impedance
($\ell=\lambda/2$)

$$Z_{in} = 73 + j42.5 \tag{4-93a}$$

Infinitesimal Dipole

$$E_\theta = C_1 \underbrace{\sin\theta}_{field\ pattern} \quad ; \quad U = C_2 \underbrace{\sin^2\theta}_{power\ pattern}$$

λ/2 Dipole

$$E_\theta = C_1' \underbrace{\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]}_{field\ pattern}; \quad U = C_2' \underbrace{\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]^2}_{power\ pattern} \cong C_2' \sin^3\theta$$